Interpretation of Finite Element Analysis Results of Scramaccelerator Thrust

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Introduction

DYNE and Heinrich[1] presented a calculation of the pressure distributions and the net thrust generated on a conical–nose projectile in the Scramaccelerator. The calculation is based on the solution of the inviscid conservation equations including finite rate chemistry by the finite element formulation. The projectile geometry is a 14-deg nose cone followed by a short cylindrical section and then by a 20-deg tail cone, shown in Fig. 1 of Ref. 1. Two projectile-to-tube-diameter ratios, 0.85 and 0.9, were used. The projectile is assumed to be fired into the tube filled with a mixture of hydrogen and oxygen at superdetonative velocity. The combustion is assumed to follow the eight-reaction six-species model used by Yungster et al.[2] The calculations were performed for projectile Mach numbers of 7, 8, and 9 at tube fill pressures of 20, 50, and 100 atm and hydrogen–oxygen equivalence ratios of 0.5, 1, and 2, respectively. The calculated variation of the thrust on the projectile as a function of its Mach number and its velocity is included in Figs. 1 and 2, respectively. The axial force acting on the front part of the projectile is evaluated by the integration of the pressure distribution acting on the front part of the projectile. Similarly, the axial force acting on the rear part of the projectile is evaluated by the integration of the pressure distribution acting on the rear part of the projectile. The difference between these axial forces is a net thrust or drag. This depends on whether the axial force on the rear of the projectile (because of combustion) is larger or smaller than the axial force on the front. The evaluations of these axial forces and the net thrust (which were presented in Figs. 5 and 6 of Ref. 1), for the case of the 0.85 projectile at a fill pressure of 50 atm are shown in this Note (see Figs. 1 and 2). It is noted in Ref. 1 that the net thrust is decreasing as the projectile velocity is increasing.

Evaluation of the Maximum Projectile Velocity from the Results of Ref. 1

To understand and draw conclusions from the very interesting results presented in Ref. 1, it is important to note the thermochemical properties of the hydrogen–oxygen mixtures, presented in Table 1. These properties are calculated and based on the Gordon and McBride code (Refs. 3 and 4).

The first observation is that the projectile must be injected into the Scramaccelerator tube at an initial velocity that is above the Chapman–Jouguet (C–J) detonation velocity of the mixture for operation in the superdetonative mode. That is an initial velocity higher than 2461.4 m/s for an equivalence ratio (ER) of 0.5, a velocity higher than 3049.2 m/s for ER = 1.0, and a velocity higher than 3591.7 m/s for ER = 2.0. These projectile velocities are well beyond the capability of powder guns but they can be achieved for small projectiles by light gas guns.

The second observation is that the net thrust is decreasing as the projectile velocity is increasing.[1] This is mainly caused by the large increase in the front axial force as the projectile speed is increasing, as seen in Figs. 1 and 2. There is also a small decrease of the rear axial force because of the combustion on the rear part, but it is very small compared with the large increase in the front axial force. It is regrettable that Ref. 1 does not present the results of the calculations for the case of ER = 0.5 and the M = 7 point for ER = 1.0. The net–thrust curves can be extrapolated to the conditions of zero thrust, as indicated in Figs. 1 and 2, using the presently available results presented in Ref. 1. At zero net thrust, the drag equals the thrust, therefore the projectile acceleration diminishes to zero and the projectile reaches its maximum velocity. The maximum Mach numbers, evaluated from Fig. 1, are 11.8 for the ER = 1.0 case and 10.3 for the ER = 2.0 case. The result of normalizing these values with the corresponding C–J detonation Mach numbers is that $M_{max}/M_{CJ} = 2.08$ for the ER = 1 case and $M_{max}/M_{CJ} = 1.89$ for the ER = 2 case. The same ratios for the corresponding velocities are obtained from Fig. 2. There, the maximum projectile velocities are 6400 and 6800 m/s for the ER = 1 and 2 cases, respectively. Normalizing by the respective detonation velocities results in the values of V_{max}/V_{CJ} = 2.08 and 1.89 for the ER = 1 and 2 cases, respectively.

The calculations of Ref. 1 neglect two very significant contributions to the drag of the projectile. First, the projectile in the Scramaccelerator must have fins for its support and guidance in its flight in the tube. The axially symmetric projectile model used in Ref. 1 does not consider the additional drag caused by the fins and the losses caused by the friction of these fins on the tube wall.

Secondly, the use of the inviscid code for the calculations of Ref. 1 results in neglecting the skin friction, and more significantly drag, because of the losses by the shock-wave/boundary-layer interactions on the cylindrical section of the projectile. The shock waves reflection patterns, shown in Figs. 3a and 3b of Ref. 1, clearly suggest that shock-wave/boundary-layer interactions can affect strongly this flowfield.

These contributions to the drag of the projectile in the tube can be expected to reduce the net thrust significantly. Therefore, the maximum velocity evaluated in Ref. 1 can be considered as the upper limit of velocity attainable in the Scramaccelerator. Including the additional drag of the fins, the fin friction losses and the viscous effects (friction drag and shock-wave/boundary-layer interactions) will reduce this maximum velocity.

It is also significant that the calculations of Ref. 1 show that the ratio of the maximum velocity to the C–J detonation velocity is about $V_{max}/V_{CJ} \cong 2$ for the different hydrogen–oxygen mixtures. It will be very interesting to see the results of the calculations for the fuel lean case, ER = 0.5. Applying the value of $V_{max}/V_{CJ} \cong 2$ to the ER = 0.5 case, the maximum velocity should be in the order of 5000 m/s, whereas the maximum Mach number should be about 10.9. Therefore, the ER = 0.5 case would show positive net thrust in the Mach number range of Fig. 1, but would probably show a negative net thrust above 5000 m/s in Fig. 2.

Examination of the Maximum Projectile Velocity Using the Results of Ref. 1 by the Energy Analysis of Ref. 5

It was shown in Ref. 5 that the maximum velocity of the projectile can be evaluated by the relation

$$\frac{V^2_{\infty_{max}}}{V^2_{CJ}} = \frac{\beta^2 \eta_p}{C_D(\gamma^2 - 1)} \tag{1}$$

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Table 1 Thermochemical properties of hydrogen–oxygen mixtures at 50 atm

Equivalence ratio	Speed of sound, m/s	C–J detonation velocity, m/s	C–J detonation Mach number	Added heat of combustion, Q/c_pT_∞
0.5	451.6	2461.4	5.45	10.72
1.0	537.7	3049.2	5.67	12.14
2.0	658.7	3591.7	5.45	10.04

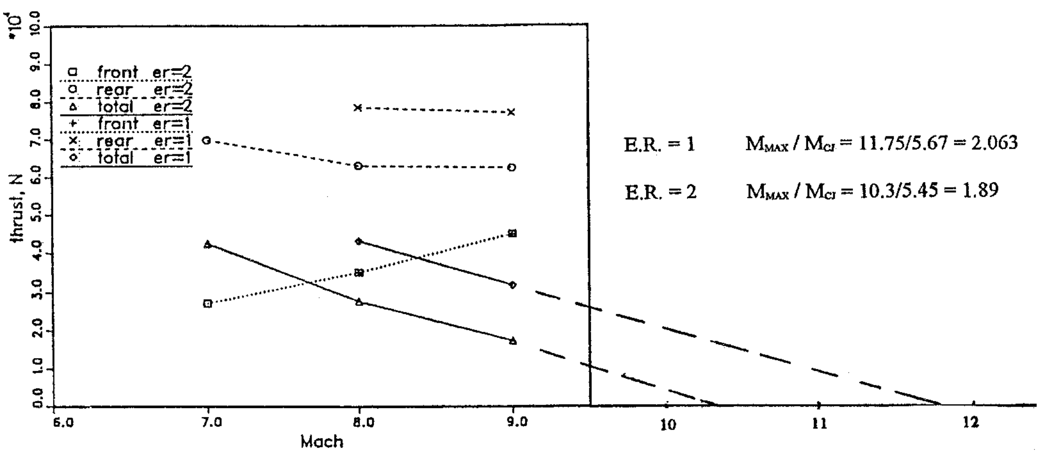

Fig. 1 Evaluation of M_{max} from the variation of the thrust vs Mach number for reacting flow through a Scramaccelerator; R = 0.85.

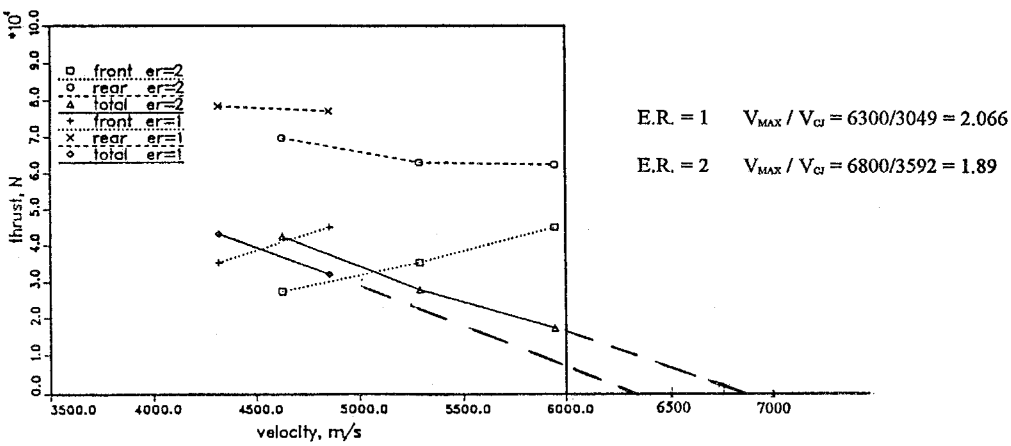

Fig. 2 Evaluation of V_{max} from the variation of the thrust vs velocity for reacting flow through a Scramaccelerator; R = 0.85.

In the case of the Scramaccelerator, β is the ratio of the tube diameter to the projectile maximum diameter, and η_p is the propulsive efficiency that is defined by

$$\eta_p = \frac{4TV_\infty}{Q\rho_0 V_\infty \beta^2 \pi d^2} \tag{2}$$

The propulsive efficiency represents the ratio of the rates of the thrust energy to the total available chemical reaction energy. The relation for the propulsive efficiency [Eq. (2)] can be expressed in terms of the thrust parameter $T/p_\infty S$ and the heat addition parameter Q/c_pT_∞

$$\eta_p = \frac{\gamma - 1}{\gamma} \frac{(T/p_\infty S)}{(Q/c_pT_\infty)\beta^2} \tag{3}$$

Using Eqs. (2) and (3), the relation for the maximum velocity of the projectile in the Scramaccelerator becomes

$$\frac{V_{max}^2}{V_{CJ}^2} = \frac{1}{2(\gamma + 1)} \frac{T}{D} \frac{M_\infty^2}{(Q/c_pT_\infty)} \tag{4}$$

The calculations of the pressure distributions in Ref. 1 enable the evaluation of the axial forces acting on the front and rear of the projectile. The results of these calculations are in Table 2. The net thrust acting on the projectile is evaluated by subtracting the front axial force from the rear axial force. The net thrust and the ratio of the maximum projectile velocity to C–J detonation velocity are shown in Table 2. The total thrust and the total drag acting on the projectile in its flight in the tube are not determined separately in the calculations of Ref. 1. It is therefore not possible to determine the propulsive efficiency or the drag coefficient of the projectile. However, it is possible to evaluate the maximum projectile velocity as indicated in Fig. 2. The values of V_{max}/V_{CJ} are indicated in Table 2. Now, using Eq. (1) it is possible to evaluate the ratio of the propulsive efficiency divided by the drag coefficient, as indicated in Table 2.

The higher value of the η_p/C_D for the ER = 1 case is supported by the calculations of Ref. 1 that indicated higher net thrust for the stoichiometric mixture compared with the fuel-lean and fuel-rich mixtures at the same flow conditions (shown in Fig. 7 of Ref. 1). It is then expected that the propulsive efficiency will be the highest for the stoichiometric mixture

Table 2 Summary of results of calculations of maximum projectile velocity of Ref. 1 with evaluation using Eq. (1)

ER	M	γ	Q/c_pT_∞	Force front, $N \times 10^4$	Force rear, $N \times 10^4$	Net thrust, $N \times 10^4$	V_{max}/V_{CJ}, Ref. 1	η_p/C_D from Eq. (1)
1	8	1.4	12.14	3.6	7.9	4.3	2.08	3.0
1	9	1.4	12.14	4.6	7.8	3.2	2.08	3.0
2	7	1.4	10.04	2.8	7	4.2	1.89	2.48
2	8	1.4	10.04	3.6	6.3	2.7	1.89	2.48
2	9	1.4	10.04	4.6	6.2	1.6	1.89	2.48

while the drag should remain the same for all mixtures. Therefore, the ratio of η_p/C_D for the stoichometric mixture should be higher than that for the fuel-rich mixture, as seen in Table 2.

The examination of the experimental data of the tests in the ram accelerator of the University of Washington indicates that the maximum velocity is about 1.15–1.2 times the C–J detonation velocity, whereas $\beta = 1.31$, as discussed in Ref. 5. The ratio of the propulsive efficiency to the drag coefficient for these ram accelerator tests, evaluated by Eq. (1), is 0.81. This value should be compared with the corresponding ratios for the Scramaccelerator, 3 for ER = 1 and 2.48 for ER = 2, as evaluated from the results presented in Ref. 1. This indicates that indeed the calculations of Ref. 1 are overoptimistic, probably because of underestimating the drag of the projectile in the tube.

Conclusions

The published results of Ref. 1 for the ER = 1 and 2 cases are in very good qualitative agreement with the energy analysis of Ref. 5. It is shown in Ref. 5 that the ratio of the maximum projectile velocity to the Chapman–Jouguet detonation velocity is determined by the ratio of the propulsive efficiency to the drag coefficient. It is shown in Ref. 6 that, based on the data of ram accelerator tests, the ratio of V_{max}/V_{CJ} is about 1.2–1.3.

The calculations of Ref. 1 indeed indicate that the maximum velocity normalized by the detonation velocity is about the same for both the hydrogen–oxygen ER = 1 and 2 cases. The maximum velocity calculated in Ref. 1 indicates an upper limit of about twice the C–J detonation velocity. By modifying the formulation of the calculations of Ref. 1 the additional drag into these calculations will reduce the ratio of the maximum velocity to the C–J detonation velocity in the Scramaccelerator. It is expected that the value of the maximum velocity will be reduced then toward $V_{max}/V_{CJ} = 1.3$.

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